A Solution Method for Linear Rational Expectation Models under Imperfect Information

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This article presents a solution algorithm for linear rational expectation models under imperfect information. "Imperfect information" means that some decisions may be made before observing some shocks, while others may be made after observing them. For example, we can consider a variant of the RBC model, in which labour supply is decided before observing today's productivity shock. In this variant, apart from the information structure (i.e., the FOC with respect to labour supply has an expectation operator), the equations that define the equilibrium are the same as in the standard RBC model. Indeed, imperfect information plays an important role in many important classes of models, such as the sticky information model of Mankiw and Reis (2001). Also, researchers often do not know *a priori* what information is available when each decision is made; they may want to estimate the information structure by parameterising it, or they may want to experiment on a model under several patterns of information structure.

Our method provides the solution of a model in the form of

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\begin{aligned}
\kappa_{t+1} &= H\kappa_t + J\xi^{t,S} \\
\phi_t &= F\kappa_t + G\xi^{t,S} \\
\xi^{t,S} &\equiv \begin{pmatrix} \xi_t^T & \cdots & \xi_{t-S}^T \end{pmatrix}^T
\end{aligned}
$$

where κ_t and ϕ_t are the vectors of crawling (predetermined) and jump variables, respectively, and ξ_{t-s} is the vector of innovations at time $t-s$, for $s = 0, \cdots, S$, where S is such that the minimum information set in the model includes all information up to time $t-S-1$. The superscript T indicates transposition, and hence $\xi^{\tau,S}$ is the vertical concatenation of $\left\{\xi_{\tau-s}\right\}_{s=0}^{S}$. H, J, F and G are the solution matrices provided by the algorithm[2].

This is obviously a generalisation of the solution form for perfect information models. First, note that imperfect information inevitably requires the expansion of the state space, which is done by the expansion of the innovation vector in our method. Second, by recording the past innovations, we can recover the information sets in past periods.

Our solution method exploits two observations; (i) if an endogenous variable $y_{k,t}$ is decided without observing an innovation $\xi_{i,t-s}$, then $y_{k,t}$ is not affected by $\xi_{i,t-s}$ (i.e., $\partial y_{k,t}/\partial \xi_{i,t-s} = 0$); (ii) if the information set in the j-th equation includes $\xi_{i,t-s}$, then $\xi_{i,t-s}$ cannot be the source of expectation error in the j-th equation ($E_{s,ji} = 0$). It turns out that these two zero restrictions are enough to pin down a unique solution.

Perhaps surprisingly, imperfect information does not alter the coefficients on the past crawling variables (i.e., H and F). Hence, if the corresponding perfect information model is saddle path stable (sunspot, explosive), an imperfect model is saddle-path stable (sunspot, explosive, respectively). Moreover, if the minimum information set contains all the information up to time $t-S-1$, then the direct effects on the impulse response functions last for only the first S periods. One such example can be found in Dupor and Tsuruga (2005).

Although imperfect information does not change the qualitative nature of a model, it can significantly alter its quantitative properties. This article demonstrates, as an example, that adding imperfect information to the RBC model remarkably improves the correlation between labour productivity and output.

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[2] The set of MATLAB codes for the general class of imperfect models is available at: http://www.kent.ac.uk/economics/research/papers/2007/0703.html