

# Spatial Patterns and Size Distributions of Cities

Wen-Tai Hsu\* Tomoya Mori,<sup>†‡</sup> Tony E. Smith<sup>§</sup>

January 2014

## Abstract

City size distributions are known to be well approximated by power laws across many countries. One popular explanation for such power-law regularities is in terms of random growth processes, where power laws arise asymptotically from the assumption of *iid* growth rates among all cities within a given country. But this assumption has additional consequences. Since all subsets of cities have the same statistical properties, each subset must exhibit essentially the same power law. Moreover, this *common power law* (CPL) property must hold regardless of the spatial relations among cities. Using data from the US, this paper shows first that spatial partitions of cities based on geographical proximity are significantly more consistent with the CPL property than are random partitions. It is then shown that this significance becomes even stronger when proximity among cities is measured in terms of trade linkages rather than simple geographical distance. These results provide compelling evidence that spatial relations between cities do indeed matter for city-size distributions. Further analysis shows that these results hinge on the natural “spacing out” property of city patterns in which larger cities tend to be widely spaced apart with smaller cities organized around them.

*JEL Classifications* : C49, R12

*Keywords* : power law, Zipf’s law, random growth, space, geography, Voronoi partition, economic region

**Acknowledgement:** For their helpful comments, we thank the seminar participants at Academia Sinica, Chinese University of Hong Kong, Kyoto University, Singapore Management University, Tokyo University, and the 2013 Annual Meeting of the Urban Economics Association. We are particularly grateful to Esteban Rossi-Hansberg for his detailed and insightful comments that helped to sharpen the focus of this work and to improve the paper in many other ways. This research has been partially supported by the Grant in Aid for Research (No. 25285074 and the Global COE Program “Raising Market Quality”) of the Ministry of Education, Culture, Sports, Science and Technology of Japan.

---

\*School of Economics, Singapore Management University. Email: wentaihsu@smu.edu.sg.

<sup>†</sup>Institute of Economic Research, Kyoto University. Email: mori@kier.kyoto-u.ac.jp.

<sup>‡</sup>Faculty fellow, Research Institute of Economy, Trade and Industry (RIETI) of Japan.

<sup>§</sup>Department of Electrical and Systems Engineering, University of Pennsylvania. Email: tesmith@seas.upenn.edu.